US-TEC
Technical Document

Based on the MAGIC code written by Paul S.J. Spencer
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This product evolved from collaboration between NOAA's
National Geodetic Survey and Space Environment Center

Introduction

US-TEC is produced by a Kalman filter based data assimilation algorithm for imaging the Earth's ionosphere in four dimensions using GPS data. The working of the algorithm also relies on third party software in the form of the IRI95 ionospheric model.

The software package on which the product is based, MAGIC, was developed by Paul Spencer and evolved from previous research by Mitchell and Spencer (2002). MAGIC is a research tool designed to study various approaches to the problem of obtaining an estimate of the ionospheric structure with sufficient accuracy to provide utility for space weather and geodetic positioning applications.

The following documentation provides a summary of the mathematical theory behind the model and information on real-time data processing.

Theory

This section provides an outline of the theory of the mathematics behind the assimilation method used by MAGIC which is based on a Kalman filter. Essentially the Kalman filter method provides a means of optimally updating a solution to a linear least squares problem by combining time dependent observations and a prior model estimate of the solution. The unknowns, which in this case represent the ionospheric electron density field, are stored in the state vector *x*. Associated with this vector is a covariance matrix, *P*, which is updated by the filter each iteration. P represents the uncertainty in the state. The sequence of steps in updating the filter may be defined as follows;

Firstly the state vector, *x*, is projected into the future (the minus superscript implies prior values)

$$x = Ax^{-} + B$$

The matrices A and B are generated from model estimates, M, as follows;

$$A_{i,j} = (1-\alpha)G_{i,j}\, M_i^{-} \Big/ M_j^{+}$$

$$B_i = \alpha M_i^{+}$$

Where the matrix G defines the correlation between spatially separated state variables, and α is a constant. The matrix G is defined by the radial, latitude and longitude covariance vectors. By default a Gaussian function is used. With α set to zero, B is zero and hence the A term only exists, which defines the forward projection in terms of the relative spatial/temporal variations in the model. With α greater than zero the B term increasingly dominates, which sets the future state estimate to that of the model in an absolute sense.

The next stage in the update of the filter involves projecting the error covariance matrix

$$P = AP^{-}A^{T} + Q$$

Where the process noise matrix Q is defined as,

$$q = (M^{+} + Ax^{-})/2$$

$$Q_{i,j} = kG_{i,j}q_i q_j$$

Where, k is a constant, and *G*, as before, defines the correlation between terms. The Q matrix therefore defines the variance as being a constant fraction of an average of the model and projected state estimate.

Given a set of line integral observations, z, with covariance R, and path integrals defined by, H, the Kalman Gain is given by

$$K = PH^{T}(HPH^{T} + R)^{-1}$$

Finally, using the Kalman gain, the state vector and its covariance are updated

$$x^{+} = x + K(z - Hx)$$

$$P^{+} = (I - KH)P$$

One final modification has been made in the form of the inclusion of linear time evolution terms. The state vector now becomes;

$$x \rightarrow \begin{bmatrix} x \\ \delta x/\delta t \end{bmatrix}$$

With terms appended to the observation matrix, H, as

$$H \rightarrow \begin{bmatrix} H & H\delta t \end{bmatrix}$$

Mapping the state using empirical ortho-normal functions

Central to the MAGIC assimilation method is the option of mapping the state vector to enable a more succinct representation of the electron density field in three-dimensions. The mapping is applied to the radial profile using a set of empirical ortho-normal functions, EOF's, obtained from the IRI95 model. The mapping significantly improves the ability of the filter to image variations in the electron density profile. As an added advantage the reduction in size of the state vector will also increase performance and reduce memory requirements by one or more orders of magnitude.

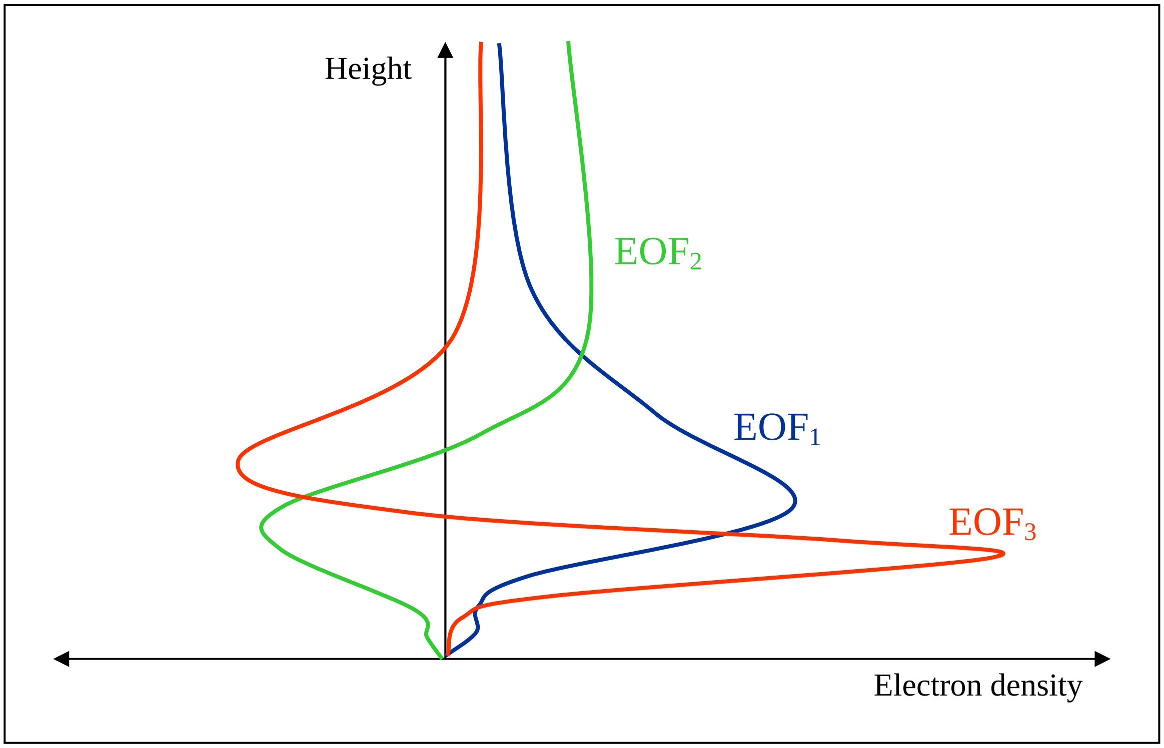

Figure 1. Example empirical ortho-normal functions (EOF's)

Example mapping functions are shown in Figure 1. These EOFS's were generated by applying singular value decomposition to a set of model profiles generated by IRI95. The dominant term, EOF_1, represents a mean ionospheric profile. The higher order EOF's, which gradually decrease in significance, allow the profile to depart from the mean. Typically 2 to 3 EOF's are sufficient when using ground-based data alone.

The mapping of the problem involves mapping all terms in the filter as shown below.

Map state vector using orthogonal mapping matrix, M

$$x \to M^T x$$

Map forward projection, A

$$A \to M^T AM$$

Map error covariance, P

$$P \to M^T PM$$

Map error process noise, Q

$$Q \to M^T QM$$

Map observation matrix, H

$$H \to MH$$

Real-time data processing

The possibility of processing data in real-time was included to fulfill the requirements of the Space Environment Center (SEC), Boulder. This software was developed specifically for the SEC and requires access to data not available to general users. This appendix provides a brief overview of the real-time system.

Cycle-slip detection and phase to pseudo-range leveling must be carried out for all available data over the past few hours for each filter update. The data is then processed into a format that can be ingested by the Kalman filter. Note that the Kalman filter state error estimates are highly dependent on the process noise parameter. One of the output products offered by the real-time software is ASCII files, containing the vertical TEC and the slant line-of-sight electron content to each satellite for a grid of points across the US mainland. An example data file is shown below. Data are stored for each satellite as a two dimensional matrix of slant TEC values (in 0.1TEC). The first row represents the longitude of the vertices and the first column the latitudes, both in units of 0.1 degrees. The value in the top left of the matrix is the satellite number plus 99900. Values larger than 99900 may be used as a marker for a new satellite. Note that values of 0 imply the satellite was below the horizon at that point or the ray to the satellite passed outside the latitude-longitude grid.

```
99905 -1300 -1290 -1280 -1270 -1260 -1250 -1240 -1230 -1220 …
  200     0     0     0     0     0     0     0     0     0 …
  210     0     0     0     0     0     0     0     0     0 …
  220     0     0     0     0     0     0     0     0     0 …
  230     0     0     0     0     0     0     0     0     0 …
  240     0     0     0     0     0     0     0     0     0 …
  250     0     0     0     0     0     0     0     0     0 …
  260     0     0     0     0     0     0     0     0   432 …
  270   516   504   493   481   470   459   449   439   429 …
  280   513   501   489   478   467   456   445   435   425 …
…
…
…
99906 -1300 -1290 -1280 -1270 -1260 -1250 -1240 -1230 -1220 …
  200   253   249   245   241   238   234   230   227   224 …
  210   241   237   234   231   227   224   221   218   216 …
```

Example ASCII slant TEC data for satellites 5 and 6

References

Mitchell, C. and P. Spencer, Development of tomographic techniques for large scale ionospheric imaging, Proceedings of the Ionospheric Effects Symposium, Alexandria, Virginia, Editor John M. Goodman, p601, 2002.

Space Environment Center
US-TEC Product
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